SIMULATION OF DELAMINATION GROWTH UNDER HIGH CYCLE FATIGUE USING COHESIVE ZONE MODELS

PEDRO P. CAMANHO*, ALBERT TURON**, JOSEP COSTA**

*DEMEGI, Faculdade de Engenharia, Universidade do Porto
Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
pcamanho@fe.up.pt

** AMADE, Polytechnic School, University of Girona
Campus Montilivi s/n, 17071 Girona, Spain
albert.turon@udg.es, josep.costa@udg.es

ABSTRACT: A cohesive zone model is proposed for the simulation of delamination growth in composite materials under high-cycle fatigue loading. The basis for the formulation is an interfacial constitutive law that links fracture mechanics and damage mechanics relating the evolution of the damage variable, **d**, with the crack growth rate, da/dN. The cohesive zone model is implemented in ABAQUS finite element code and used in the simulation of carbon-epoxy test specimens cyclically loaded in mode I, mode II and mixed-mode I and II. The accuracy of the model is assessed by comparing the predictions with experimental data.

Keywords: Composite materials; delamination; fatigue; cohesive models.

RESUMO: Este artigo apresenta um modelo coesivo para simular a delaminagem de materiais compósitos sob fadiga de altos ciclos. O modelo constitutivo proposto relaciona a evolução da variável de dano, **d**, com a velocidade de crescimento da delaminagem, da/dN. O modelo coesivo é implementado no código de elementos finitos ABAQUS e é utilizado na simulação de provetes fabricados em carbono-epoxy carregados ciclicamente em modo I, modo II e modo misto I e II. O modelo é validado comparando as suas previsões com resultados experimentais.

Palavras chave: Materiais compósitos; delaminagem; fadiga; modelos coesivos.

1. INTRODUCTION

High-cycle fatigue is a common cause of failure of aerospace structures. In laminated composite materials, the fatigue process involves several damage mechanisms that result in the degradation of the structure. One of the most important fatigue damage mechanisms is interlaminar damage (delamination).

In a degradation process involving high cycle-fatigue, damage evolution can be obtained as the sum of the damage caused by static or quasi-static overloads and the damage that result from the cyclic loads. The damage evolution produced by cyclic loads is usually represented as a function of the number of cycles and strains (or displacement jumps) [1-3], where a damage evolution law expressed in terms of the number of cycles is established a priori. However, the damage evolution law must be expressed as a function of several parameters that have to be adjusted through a trial-and-error calibration of the whole numerical model. In this paper, an alternative approach is proposed whereby the evolution of damage is based on linking Fracture Mechanics and Damage Mechanics, and relating the evolution of the damage variable, **d**, with the crack growth rate, da/dN.

The present model is implemented by means of a user-written element in ABAQUS [4] by adding the damage evolution law formulated in the cohesive element previously developed by the authors [5].

2. CONSTITUTIVE MODEL FOR HIGH-CYCLE FATIGUE

The damage evolution in a degradation process involving high-cycle fatigue can be considered as the sum of the damage caused by the quasi-static loads and the damage caused by the cyclic loads:

$$\frac{d\mathrm{d}}{dt} = \dot{\mathrm{d}} = \dot{\mathrm{d}}_{static} + \dot{\mathrm{d}}_{cyclic} \tag{1}$$

The evolution law is formulated using Fracture Mechanics and creating a link between Fracture Mechanics and Damage Mechanics to relate the damage variable, **d**, to the crack growth rate, da/dN. The evolution of the damage variable is related to the evolution of the crack surface as follows:

$$\frac{\partial \mathbf{d}}{\partial N}=\frac{\partial \mathbf{d}}{\partial A_{\mathbf{d}}}\frac{\partial A_{\mathbf{d}}}{\partial N} \tag{2}$$

where $A_{\mathbf{d}}$ is the damaged area, and $\frac{\partial A_{\mathbf{d}}}{\partial N}$ is the growth rate of the damaged area. While the second term in the right hand side of equation (2) must be characterized experimentally, the first term, $\frac{\partial \mathbf{d}}{\partial A_{\mathbf{d}}}$, can be obtained from either a Damage Mechanics approach or a Fracture Mechanics approach.

In the framework of the Fracture Mechanics, the fraction of the damaged area, $A_{\mathbf{d}}$, with respect to the total area, A, can be written as a function of the dissipated energy:

$$\frac{A_{\mathbf{d}}}{A}=\frac{\Xi}{G_c} \tag{3}$$

where Ξ is the fraction of the energy per unit surface dissipated during the damage process, i.e., the area under the cohesive law for the current damage threshold, and G_c is the critical energy release rate.

Assuming no change between modes, G_c is constant, while Ξ is a function of the cohesive law used and the current damage threshold. Using this approach, the derivative of the damage variable with respect to the damaged area can be written as:

$$\frac{\partial \mathbf{d}}{\partial A_{\mathbf{d}}}=\frac{G_c}{A}\frac{\partial \mathbf{d}}{\partial \Xi} \tag{4}$$

The derivative of the energy dissipation with respect to the damage variable is obtained from the equations of the constitutive law used. Using a bilinear constitutive law:

$$\frac{\partial \Xi}{\partial \mathbf{d}}=G_c\frac{\Delta^0\left(\Delta^f-\Delta^0\right)}{\left(\Delta^f\left(1-\mathbf{d}\right)+\mathbf{d}\Delta^0\right)^2} \tag{5}$$

Using equation (5), equation (4) can be written as:

$$\frac{\partial \mathbf{d}}{\partial A_{\mathbf{d}}}=\frac{1}{A}\frac{\left(\Delta^f\left(1-\mathbf{d}\right)+\mathbf{d}\Delta^0\right)^2}{\Delta^0\left(\Delta^f-\Delta^0\right)} \tag{6}$$

For a specimen with just one crack front, the crack growth rate is equal to the sum of the damaged surface growth rates of all elements in the cohesive zone. In the other regions of the specimen, there is no possibility of new surface generation.

$$\frac{\partial A}{\partial N}=\sum_{e\in A_{CZ}}\frac{\partial A_{\mathbf{d}}^e}{\partial N} \tag{7}$$

Using the simplification that $\frac{\partial A_{\mathbf{d}}^e}{\partial N}$ is constant over the cohesive zone, the previous equation can be written as:

$$\frac{\partial A}{\partial N}=\sum_{e\in A_{CZ}}\frac{\partial A_{\mathbf{d}}^e}{\partial N}=\frac{A_{CZ}}{A}\frac{\partial A_{\mathbf{d}}}{\partial N} \tag{8}$$

where the ratio $\frac{A_{CZ}}{A}$ represents the number of areas in which the cohesive zone has been divided. In a finite element environment, this ratio represents the number of elements that span the cohesive zone. Rearranging terms in equation (8), the surface damage growth rate can be written as:

$$\frac{\partial A_{\mathbf{d}}}{\partial N}=\frac{A}{A_{CZ}}\frac{\partial A}{\partial N} \tag{9}$$

Using equations (4) and (9) in (2):

$$\frac{\partial \mathbf{d}}{\partial N}=\frac{G_c}{A_{CZ}}\frac{\partial \mathbf{d}}{\partial \Xi}\frac{\partial A}{\partial N}=\frac{bG_c}{A_{CZ}}\frac{\partial \mathbf{d}}{\partial \Xi}\frac{\partial a}{\partial N} \tag{10}$$

where G_c depends on the material used and loading mode, and $\frac{\partial \mathbf{d}}{\partial \Xi}$ depends on the cohesive law used in the formulation of the surface traction-displacement jump relation.

The area of the cohesive zone can be computed using:

$$A_{CZ}=\eta\frac{E_3 G}{\left(\tau^o\right)^2}b \tag{11}$$

where b is the width of the specimen, G is the energy release rate, E_3 is the Young's modulus of the bulk material in the direction perpendicular to the crack plane, and τ^o is the interfacial strength. The parameter η depends on the softening law and geometry of the specimen, being only a material property equal to $\pi/4$ for very large geometries.

3. CRACK GROWTH RATE

The crack growth rate under fatigue loading, $\frac{\partial a}{\partial N}$, is a load and material-dependent characteristic that has been widely studied. The crack growth rate of brittle composites may be expressed using Paris law:

$$\frac{\partial a}{\partial N}=C\left(\frac{\Delta G}{G_c}\right)^m \tag{12}$$

where C and m are parameters that depend on the mode ratio and must be determined experimentally. ΔG, is the cyclic variation in the energy release rate, which can be computed using the constitutive law of the interface:

$$G = \int_0^{\Delta} \tau(\Delta) d\Delta \tag{13}$$

Defining the reversibility factor, R, the relation between the minimum and the maximum displacement is:

$$\Delta^{\min} = R\Delta^{\max} \tag{14}$$

Using (13) and (14):

$$\Delta G = \frac{K\Delta^0 \Delta^{\max}\left(\Delta^f - \Delta^{\max}\dfrac{1+R}{2}\right)}{\Delta^f - \Delta^0}(1-R) \tag{15}$$

4. MODEL VALIDATION

The proposed model was implemented in ABAQUS (4) as a user-defined element. The preliminary validation of the model was performed by comparing its predictions with published experimental data.

The Double-Cantilever Beam specimen tested by Asp [6] was simulated to analyze mode I fatigue delamination. The relevant material properties used in the model are reported by Asp [6]. The test specimen is 20mm wide, 150mm long, with two 1.55mm thick arms with an initial crack of 35mm. The specimen is loaded by constant moments to ensure a constant value of the energy release rate, regardless of the crack length [2]. This model is required to obtain the crack growth rates for different values of the applied energy release rate.

Applying constant moments to the specimen, the energy release rate is related to the moment as:

$$G_I = \frac{M^2}{bEI} \tag{16}$$

where E is the longitudinal flexural Young's modulus, and I is the second moment of area of the specimen's arm.

The application of constant moments to the arms of the DCB specimen results in a linear relation between the crack length and the number of cycles. For example, the relation between the crack growth and the number of cycles is shown in Figure 1 for a ratio of 40% between the applied energy release rate and the fracture toughness.

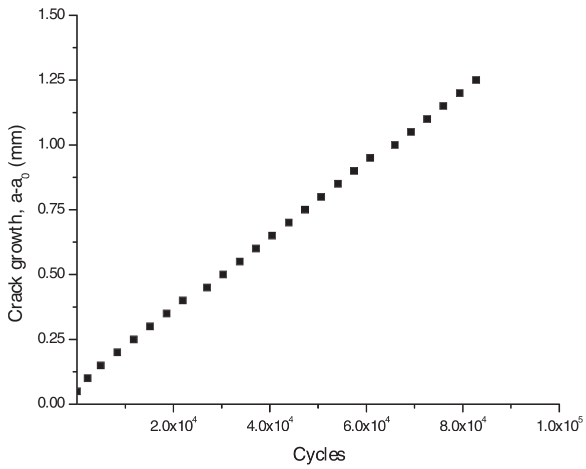

Fig. 1. Predicted relation between the crack length and the number of cycles.

Several simulations corresponding to different levels of the applied energy release rate for four different load ratios were conducted to simulate the crack growth under mode I loading.

The results obtained and the experimental data obtained by Asp et al. [6] are compared in Figure 2. It is observed that the constitutive model accounts for all three regions of fatigue crack growth. In region II, where crack growth rates follow the Paris Law, a good agreement between the predictions and the experimental data is obtained. In region I there is negligible crack growth rate for small values of the normalized energy release rate and the numerical data follows the trend of the experimental data. A difference between the numerical and the experimental data is observed in region III. One of the reasons for this difference is that the crack growth rates present in region III are very high. Therefore, a low-cycle instead of a high-cycle fatigue model is likely to be more appropriate for this region. However, in spite of this difference, the model can also predict region III crack growth rate, where the Paris law equation is not valid.

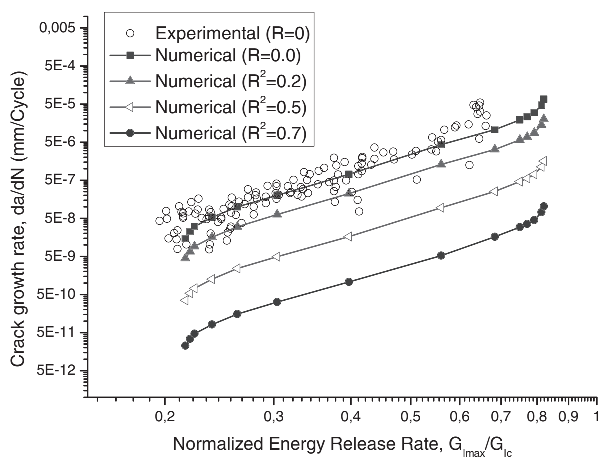

Fig. 2. Comparison of the experimental data with the predicted crack growth rate; sensitivity to the load ratio under mode I loading.

Several tests were conducted to simulate the crack growth rate under mode II loading for different ranges of the energy release rate. The experimental data on fatigue driven delamination growth reported in Asp et al. [6] was selected for comparison. The dimensions and the material of the specimen are the same used for the DCB specimen previously described. To impose pure mode II loading conditions, the four-point end notched flexure (4ENF) test specimen was used.

In the 4ENF specimen, the energy release rate is related to the applied moment, $\frac{cP}{2}$, as [2]:

$$G_{\mathrm{II}} = \frac{3}{4}\frac{(\frac{cP}{2})^2}{bEI} \tag{17}$$

The finite element model used is similar to that used in the simulation of the mode I test, the only differences being the boundary conditions and loads. The material properties used in the simulations are given by Asp [6].

The crack growth rates obtained from the different simulations and the experimental data selected for comparison are shown in Figure 3. A good correlation between the experimental data and the numerical predictions is observed.

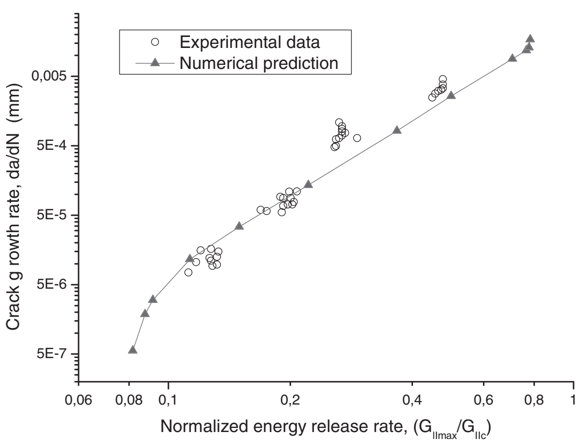

Fig. 3. Comparison of the experimental data with the crack growth rate obtained from the numerical simulation for a mode II 4ENF test.

Numerical models were developed to simulate the crack growth rate under mixed-mode loading with $G_{\mathrm{I}} = G_{\mathrm{II}}$ for different energy release rates. The experimental data on fatigue driven delamination growth reported in [6] was selected for comparison. The dimensions and the material of the specimen are the same used for the DCB specimen previously described.

In the standard mixed-mode bending test, the applied energy release rate changes with the crack length. To obtain an energy release rate independent of the crack length, the boundary conditions of the FEM were introduced using different moments applied to the arms of the specimen. The energy release rate is related to the applied moment, M, as [2]:

$$G_{\mathrm{I}} = G_{\mathrm{II}} = \frac{3}{4\left(1+\frac{\sqrt{3}}{2}\right)^2}\frac{M^2}{bEI} \tag{18}$$

The finite element model used was similar to that used in the simulation of the DCB test, where the moments applied to the arms of the specimen were modified to impose mixed-mode loading. The load is applied in two steps and the material properties used in the simulations are given [6].

The results obtained from the simulations and the experimental data are shown in Figure 4. Like in the examples of specimens loaded under pure mode I and mode II, a good correlation predictions and experiments is observed.

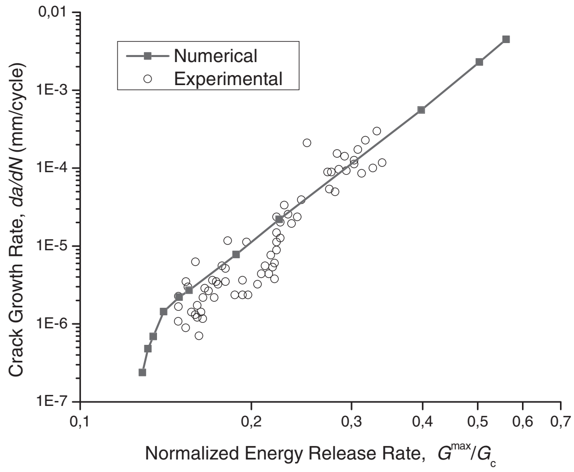

Fig. 4. Comparison of the experimental data with the crack growth rate obtained from the numerical simulation for a mixed-mode test with $G_I = G_{II}$.

5. CONCLUSIONS

A thermodynamically consistent damage model for high-cycle fatigue delamination was developed. The evolution of the damage variable was derived by linking Fracture Mechanics and Damage Mechanics to relate damage evolution to crack growth rates. The damage evolution laws for cyclic fatigue were combined with the law of damage evolution for quasi-static loads within a cohesive element previously developed by the authors. The model was validated by comparing its predictions with published experimental data. The model was able to reproduce the test data without the need of additional adjustment parameters that are typically used in other fatigue growth models.

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